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Backstepping linearization controller of the Delta Wing Rock Phenomena

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ABSTRACT
This paper deals with the control of the wing rock phenomena of a delta wing aircraft. A control technique is proposed to stabilize the system. The controller is a BACKSTEPPING controller. It is appeared that the proposed solution of control guarantee the asymptotic convergence to zero of all the states of the system. To show the performance of the proposed controller, simulation results are presented and discussed. It is found that the control scheme work well for the wing rock phenomena of a delta wing aircraft.
Key words: Wing Rock, Nonlinear Control of Wing Rock, Back stepping Control

I. INTRODUCTION

Wing rock motion is a self-induced, limit-cycle rolling motion experienced by flight aircrafts with small aspect-ratio wings, or with long pointed forebodies at high angles of attack [1]. This phenomenon has been studied by many researchers, (see for example [1],[3],[4]) because of its importance in the stability of an aircraft during high angle of attack maneuvers. It was also reported in [6] that the oscillation that does not have a limit cycle can happened at an 80/65 degree double delta wing.

Wing rock is a nonlinear phenomenon experienced by aircraft in which oscillations and unstable sideslip behavior are experienced [9]. This instability may diminish flight effectiveness or even present a serious danger due to potential instability of the aircraft [1]. Wing rock has been extensively studied experimentally, resulting in mathematical models that describe the nonlinear rolling motion using simple differential equations as in [7],[8].

The wing rock model for a delta wing aircraft used in [1] is considered in this project. Wing rock is usually modeled as self-induced, pure rolling motion, which causes the rolling moment to be a nonlinear function of the roll angle ϕ and the roll-rate p. The coefficients of such nonlinear function are obtained by curve fitting with experimental data at specific values of angle of attack. In addition, yawing dynamic is added to the nonlinear function by considering the yawing rate r = - ($\partial\beta/\partial t$) and ignoring the nonlinear term involving β due to its small value compared with the other nonlinear terms. The wing rock motion is illustrated in figure 0.

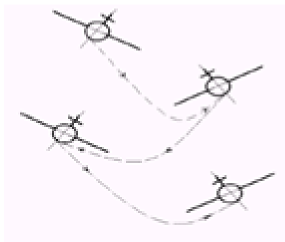


Figure 0. Wing Rock motion

II. MODEL OF THE WING ROCK PHENOMENON

Define the following variables:

ϕ: Bank angle "roll angle"

p =: Roll-rate (rad./s) ($p = \partial\phi/\partial t$).

δ: Aileron angle.

β: Sideslip angle.

$\frac{\partial\beta}{\partial t}$: Sideslip rate of change.

The differential equations describing the wing rock phenomenon are obtained using experiments and data curve fitting, such that [1]:

The rolling moment is described by the following differential equation:

$$\frac{\partial p}{\partial t} = \mu p + f(\phi, p) + L_\beta \beta + L_\delta \delta \quad \textbf{(2.1)}$$

where μ is the sting damping coefficient, L_β , L_δ are parameters.

The yawing moment is described by the following differential equation:

$$\frac{\partial^2 \beta}{\partial t^2} = -N_\beta \beta + N_r (\frac{\partial\beta}{\partial t}) - N_p p \quad \textbf{(2.2)}$$

where N_β, N_r, N_p are parameters.
The differential equation for the first order aileron actuator is taken to be:

$$\partial\delta / \partial t = (u-\delta)/\tau \quad \textbf{(2.3)}$$

where τ is the actuator time, and *u* is the controller.

The nonlinear self-induced rolling function $f(\phi, p)$ using five terms curve-fit [1] as follows:

$$f(\phi, p) = a_1\phi + a_2 p + a_3 p^3 + a_4\phi^2 p + a_5\phi p^2 \quad \textbf{(2.4)}$$

where coefficients a_1, a_2, a_3, a_4, a_5 are dependent on the angle of attack, taken to be 0.2 radian.

If the state variables are denoted by: $x = (\phi,\ p,\ \delta,\ \beta,\ \partial\beta/\partial t)^T$ then the state equations can be written as follows:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= -\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t) \\
&\quad + L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t) \\
\dot{x}_3(t) &= -k x_3(t) + ku \\
\dot{x}_4(t) &= x_5(t) \\
\dot{x}_5(t) &= -N_p x_2(t) - N_\beta x_4(t) - N_r x_5(t)
\end{aligned} \quad \textbf{(2.5)}$$

The parametric values for the aerodynamics are

Table 1: parametric values

a_1	-0.05686
a_2	0.03254
a_3	0.07334
a_4	-0.3597
a_5	1.4681
μ_1	0.354* a_2 -0.001
μ_2	0.354* a_3
b_1	0.354* a_4
b_2	0.354* a_5
ω^2	0.354* a_1
L_δ	1
L_β	-0.02822
L_r	0.1517
k	1/0.0495
N_p	-0.0629
N_β	1.3214
N_r	-0.2491

As an oscillating system, the dynamics of wing rock phenomenon with no control will be unstable and oscillating with limit cycle motion. The unstable behavior on the aircraft's wings appears with undesirable yawing motion in the flight, which might cause serious damage. To see such instable oscillating dynamics of the phenomenon, we can plot the states with no control (u =0). Figure 1 – Figure 5 show the plots of

$\phi,\ p,\ \delta,\ \beta,\ \frac{\partial\beta}{\partial t}$ respectively.

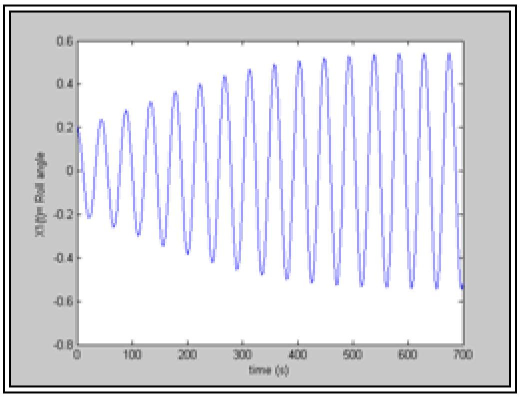


Figure 1: $\phi =$ roll angle (rad.)

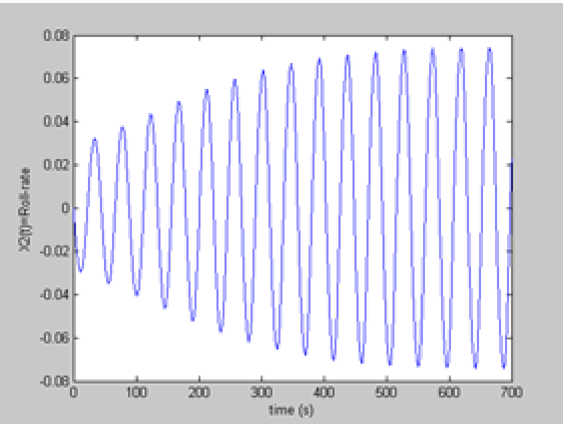


Figure 2 p = roll-rate (rad./s)

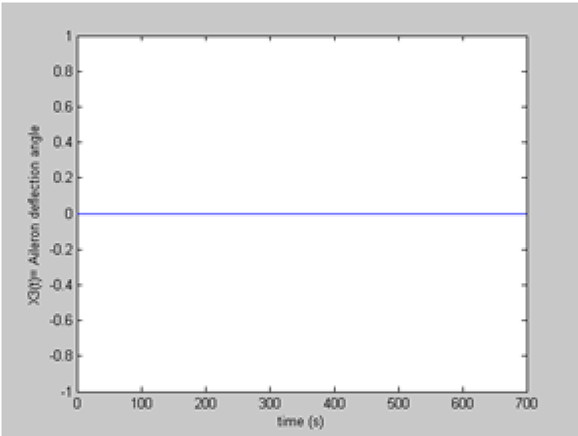


Figure 3 δ = aileron angle (rad.)

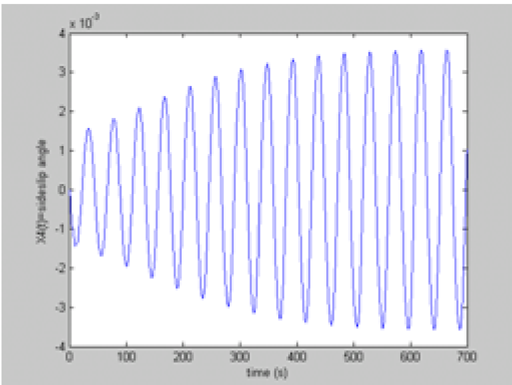


Figure 4 β sideslip angle (rad.)

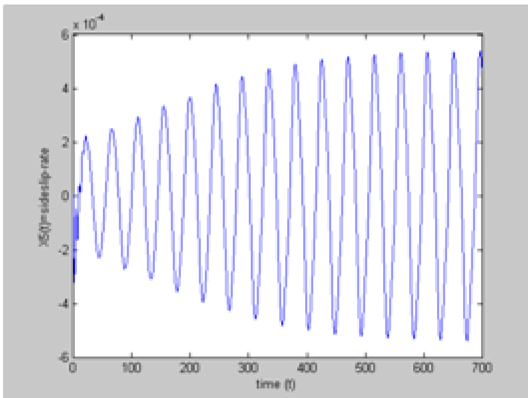


Figure 5 $\partial\beta / \partial t$ sideslip rate of change (rad/s)

It is clear from figures 1-5 that the dynamics of the system is oscillatory. Notice that the aileron deflection angle is zero because the input is zero (with no control), and zero initial value of the x (3) state.

The obtained limit cycle response in undesirable from weapon aiming and delivery considerations, and also because it may lead to structural damage in the aircraft (or other mechanical systems) thereby causing the wings to come off.

The next plots show the phase plots, x(2) versus x(1) and x(5) versus x(4). Such plots show the boundaries of the limit cycle motion of the system.

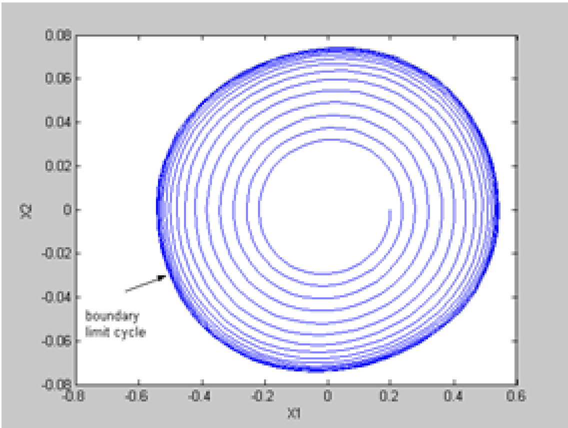


Figure 6 Roll rate vs. Roll angle

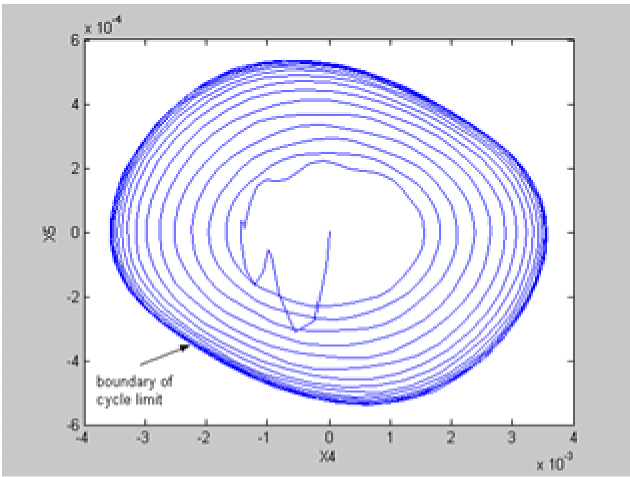


Figure 7 sideslip Rate vs. Sideslip angle

Transformation Function T (x)

The dynamic of the wing rock phenomenon is highly nonlinear. Therefore, a nonlinear transformation $z = T(x)$ [2] will be used to transfer the dynamic model of the system into a form that will simplify the design of nonlinear control schemes.

The transformation $z = T(x)$ is defined such that:

$$z_1 = N_p x_1 + N_r x_4 + x_5$$

$$z_2 = -N_\beta x_4$$

$$z_3 = -N_\beta x_5 \tag{2.6}$$

$$z_4 = N_\beta (N_p x_2 + N_\beta x_4 + N_r x_5)$$

$$z_5 = N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^2(t) x_2(t) + b_1 x_2^3(t) + b_2 x_1(t) x_2^2(t)$$

$$+ L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)$$

The inverse transformation $x = T^{-1}(z)$ exists and it is as follows.

$$x_1 = \frac{1}{N_p}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)$$

$$x_2 = \frac{1}{N_\beta N_p}(z_4 + N_\beta z_2 + N_r z_3)$$

$$x_3 = \frac{1}{L_\delta}[z_5 + (\omega^2 N_\beta - \frac{b_2}{N_\beta N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)]$$
$$- \frac{1}{L_\delta}[(\mu_1 + \frac{\mu_2}{N_p^2}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)^2 + \frac{b_1}{N_\beta^2 N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_4 + N_\beta z_2 + N_r z_3)]$$
$$+ \frac{1}{L_\delta}[N_p L_\beta z_2 - N_p L_r z_3 + N_\beta z_3 + N_r z_4]$$

$$x_4 = -\frac{1}{N_\beta} z_2 \tag{2.7}$$

$$x_5 = -\frac{1}{N_\beta} z_3$$

Hence, the dynamic model of the wing rock phenomenon can be written as,

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4 \qquad \textbf{(2.8)}$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = q(x) + g(x)u$$

where:

$$q(x) = \left[N_\beta N_p(-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t) + + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r(N_p x_2 + N_\beta x_4 + N_r x_5) \right] - L_\delta k\, x_3(t)$$

$$g(x) = kN_\beta N_p L_\delta$$

It can be seen that the model given by (2.8) has a form that simplifies the design of different nonlinear controllers. There are many teqqnuqes that deals with wing rock phenomenon like fuzzy control [10], [11], neural-network control [12] and state feed back control [13] whrer in this project different control algorithms will be designed for the wing rock phenomenon using the model given by (2.8), then using the transformation (2.6) –(2.7) to convert the designed control schemes to the original domain.

III. BACKSTEPPING CONTROLLER FOR THE WING ROCK PHENOMENON

Backstepping design is a powerful tool for designing controllers because it is a systematic technique. The idea of the backstepping design is to divide the system into sub-systems and solve for controllers for each subsystem, the final controller is used to control the whole system. A backstepping controller for the wing rock phenomenon is designed in this chapter. The system under consideration has five state equations so the first sub-system to design a controller for is the first equation, then the second controller uses both the first and the second equations and so on until all the states of the system are included.

3.1 Preliminaries about Backstepping design

Consider a system of the following form:

$$\dot{\eta} = f(\eta) + g(\eta)\zeta$$

$$\dot{\zeta} = u$$

Assume the system can be stabilized by a state feedback controller such that:

$$\zeta = \phi(\eta) \to (\phi(0) = 0)$$

That means

$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta)$ is asymptotically stable. Let a lyaponuv function be $V(\eta)$ such that:

$$\dot{V}(\eta) = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial \eta} \otimes \frac{\partial \eta}{\partial t} = \frac{\partial V}{\partial \eta} \otimes \dot{\eta} = \frac{\partial V}{\partial \eta}(f + gu) \le -w(\eta)$$

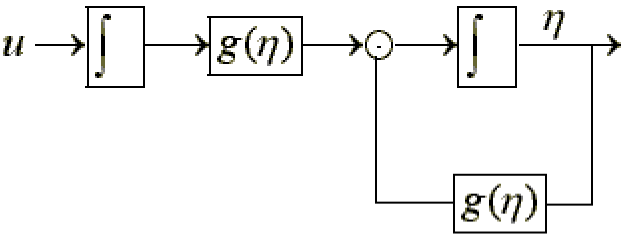
where W is positive definite

The system can be Rewritten such that:

$$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta) + g(\eta)[\zeta - \phi(\eta)]$$

Now let $z = \zeta - \phi(\eta)$, then system can be rewritten as:

$\dot{\eta} = [f + g\phi] + gz \;, \dot{z} = u - \dot{\phi} = v$



3.2 Design of the Feedback linearization Controller

Recall that the wing rock model can be written in the transformed domain as follows:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4 \qquad \textbf{(3.1)}$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = q(x) + g(x)u$$

Define **sub-system 1** as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= u_1 \end{aligned} \qquad \textbf{(3.2)}$$

System (3.2) can be written in compact form as:

$$\dot{z}_1 = f_1 + g_1 u \qquad \textbf{(3.3)}$$

Let the first controller be:

$$z_2 = -z_1 = \phi_1(z) \qquad \textbf{(3.4)}$$

Define ξ_1 such that

$$\xi_1 = z_2 + z_1 \qquad \textbf{(3.5)}$$

The closed loop for sub-system 1 is:

$$\dot{z}_1 = -z_1 \qquad \textbf{(3.6)}$$

It is obvious that the above system is stable, we can use the following lyaponuv function:

$$V_1 = \frac{1}{2} z_1^2 \qquad \textbf{(3.7)}$$

The time derivative of V_1 is as follows:

$$\dot{V}_1 = z_1 \dot{z}_1 = -z_1^2 \le 0 \qquad \textbf{(3.8)}$$

The first controller which will be the basic controller for the coming design can be evaluated using a derived equation for the backstepping controller such that:

$$u_1 = \frac{\partial \phi_1}{\partial z}[f_1 + g_1(\phi_1(z))] - \frac{\partial V_1}{\partial z} * g_1 - k[\xi_1] = -(k+1)(z_2 + z_1) \quad \textbf{(3.9)}$$

where k is a positive scalar.

Now we can prove the stability of sub-system 1 by using the following Lyaponuv function:

$$V_{a_1} = V_1 + \frac{1}{2}[z_2 + z_1]^2 \quad \textbf{(3.10)}$$

Define **sub-system 2** as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_2 &= u_2 \end{aligned} \quad \textbf{(3.11)}$$

Sub-system (3.11) can be written in compact form as:

$$\dot{z}_2 = f_2 + g_2 u \quad \textbf{(3.12)}$$

Let the second controller be equal to the controller (3.9) such that,

$$z_3 = u_1 = -(k+1)(z_2 + z_1) = \phi_2(z) \quad \textbf{(3.13)}$$

Define ξ_2 such that

$$\xi_2 = z_3 - u_1 = z_3 + (k+1)(z_2 + z_1) \quad \textbf{(3.14)}$$

The closed loop for sub-system 2 is:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= u_1 \\ \dot{z}_2 &= u_2 \end{aligned} \quad \textbf{(3.15)}$$

Consider the following Lyaponuv function:

$$V_2 = V_{a_1} = V_1 + \frac{1}{2}[z_2 + z_1]^2 = \frac{1}{2} z_1^2 + \frac{1}{2}[z_2 + z_1]^2 \quad \textbf{(3.16)}$$

The second controller will be evaluated as the follows:

$$u_2 = \frac{\partial \phi_2}{\partial z}[f_2 + g_2(\phi_2(z))] - \frac{\partial V_2}{\partial z} * g_2 - k[\xi_2]$$

$$\textbf{(3.17)}$$

$$= -(k+1)(z_2 + z_3) - (z_2 + z_1)(1 + k(k+1)) - kz_3$$

The stability can be proven for the second sub-system by using the following Lyaponuv function:

$$V_{a_2} = V_2 + \frac{1}{2}[\xi_2]^2 = \frac{1}{2} z_1^2 + \frac{1}{2}[z_2 + z_1]^2 + \frac{1}{2}[z_3 + (k+1)(z_2 + z_1)]^2 \quad \textbf{(3.18)}$$

Define **sub-system 3** as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= u_3 \end{aligned} \quad \textbf{(3.19)}$$

Sub-system (3.19) can be written in compact form as:

$$\dot{z}_3 = f_3 + g_3 u \tag{3.20}$$

Let the third controller to be as follows:

$$z_4 = u_2 = -(k+1)(z_2+z_3) - (z_2+z_1)(1+k(k+1)) - kz_3 = \phi_3(z) \tag{3.21}$$

Define ξ_3 such that

$$\xi_3 = z_4 - u_2 = z_4 + (k+1)(z_2+z_3) + (z_2+z_1)(1+k(k+1)) + kz_3 \tag{3.22}$$

The closed loop system is:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= u_2 \\ \dot{z}_4 &= u_3 \end{aligned} \tag{3.23}$$

The following Lyaponuv function is considered for sub-system 3:

$$V_3 = V_{a_2} = \frac{1}{2}z_1^2 + \frac{1}{2}[z_2+z_1]^2 + \frac{1}{2}[z_3 + (k+1)(z_2+z_1)]^2 \tag{3.24}$$

The third controller is evaluated as follows:

$$u_3 = \frac{\partial \phi_3}{\partial z}[f_3 + g_3(\phi_3(z))] - \frac{\partial V_3}{\partial z} * g_3 - k[\xi_3]$$

$$= -(k^2+k+1)z_2 - (2k+3)z_3 - (2k+1)z_4 - [z_3 + (k+1)(z_2+z_1)] \tag{3.25}$$

$$-k[z_4 + (k+1)(z_2+z_3) + (z_2+z_1)(1+k(k+1)) + kz_3]$$

The Lyaponuv equation that proves the stability of sub-system 3 is:

$$V_{a_3} = V_3 + \frac{1}{2}[\xi_3]^2 = \frac{1}{2}z_1^2 + \frac{1}{2}[z_2+z_1]^2 + \frac{1}{2}[z_3 + (k+1)(z_2+z_1)]^2$$
$$+\frac{1}{2}[z_4 + (k+1)(z_2+z_3) + (z_2+z_1)(1+k(k+1)) + kz_3]^2 \tag{3.26}$$

Define **sub-system 4** as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= z_5 \\ \dot{z}_5 &= u_4 = q(x) + g(x)u \end{aligned} \tag{3.27}$$

where:

$$q(x)=[N_\beta N_p(-\omega^2 x_1(t)+\mu_1 x_2(t)+\mu_2 x_1{}^2(t)x_2(t)+b_1 x_2{}^3(t)+b_2 x_1(t)x_2{}^2(t)++L_\beta x_4(t)-L_r x_5(t))$$
$$+N_\beta^2 x_5-N_\beta N_r(N_p x_2+N_\beta x_4+N_r x_5)]-L_\delta k\, x_3(t)$$

$$g(x)=kN_\beta N_p L_\delta$$

Sub-system (4.27) will be written in compact form as:

$$\dot{z}_4 = f_4+g_4 u \tag{3.28}$$

Let the fifth controller be as follows:

$$z_5=u_3=-(k^2+k+1)z_2-(2k+3)z_3-(2k+1)z_4-[z_3+(k+1)(z_2+z_1)] \tag{3.29}$$

$$-k[z_4+(k+1)(z_2+z_3)+(z_2+z_1)(1+k(k+1))+kz_3]=\phi_4(z)$$

Define ξ_4 such that

$$\xi_4=z_5-u_3=z_5+(k^2+k+1)z_2+(2k+3)z_3+(2k+1)z_4+[z_3+(k+1)(z_2+z_1)]$$
$$+k[z_4+(k+1)(z_2+z_3)+(z_2+z_1)(1+k(k+1))+kz_3] \tag{3.30}$$

The closed loop system is:

$$\begin{aligned}\dot{z}_1&=z_2\\ \dot{z}_2&=z_3\\ \dot{z}_3&=z_4\\ \dot{z}_4&=u_3\\ \dot{z}_5&=u_4\end{aligned} \tag{3.31}$$

The following Lyaponuv function is considered for the stability of sub-system 4:

$$V_4=V_{a_3}=\frac{1}{2}z_1^2+\frac{1}{2}[z_2+z_1]^2+\frac{1}{2}[z_3+(k+1)(z_2+z_1)]^2$$
$$+\frac{1}{2}[z_4+(k+1)(z_2+z_3)+(z_2+z_1)(1+k(k+1))+kz_3]^2 \tag{3.32}$$

The fourth controller is evaluated as follows:

$$u_4=\frac{\partial\phi_4}{\partial z}[f_4+g_4(\phi_4(z))]-\frac{\partial V_4}{\partial z}*g_4-k[\xi_4]=-[k^3+k^2+2k+1]z_2$$

$$-[k^3+3k^2+4k+2]z_3-[2k^2+3k+4]z_4-[3k+1]z_5-[z_4+(k+1)(z_2+z_3)$$

$$+(z_2+z_1)(1+k(k+1))+kz_3]-k[z_5+(k^2+k+1)z_2+(2k+3)z_3+(2k+1)z_4 \tag{3.33}$$

$$+[z_3+(k+1)(z_2+z_1)]+k[z_4+(k+1)(z_2+z_3)+(z_2+z_1)(1+k(k+1))+kz_3]]$$

The equation above can be simplified such that:

$$\therefore u_4 = -[k^3 + k^2 + 2k + 1]z_2 - [k^3 + 3k^2 + 4k + 2]z_3 - [2k^2 + 3k + 4]z_4 - [3k + 1]z_5 - [\xi_3] - k[\xi_4] \quad \textbf{(3.34)}$$

Hence, we obtain:

$$u_4 = -\psi_1 z_1 - \psi_2 z_2 - \psi_3 z_3 - \psi_4 z_4 - \psi_5 z_5 \quad \textbf{(3.35)}$$

with,

$$\psi_1 = [(k^2 + k + 1)(k^2 + 1) + k + k^2]$$
$$\psi_2 = [(k^3 + k^2 + 2k + 1) + (2k + 1)]$$
$$\psi_3 = [(k^3 + 3k^2 + 4k + 2) + (2k + 1) + k(2k + 3) + k + k^2(2k + 1)]$$
$$\psi_4 = [(2k^2 + 3k + 4) + 1 + k(2k + 1) + k^2]$$
$$\psi_5 = [4k + 1]$$

The nonlinearities of the system need to be canceled out. Therefore, the final or global controller for the wing rock phenomenon is such:

$$\begin{aligned} u &= \frac{1}{g(x)}[-q(x) + u_4] \\ &= \frac{1}{g(x)}[-q(x) - \psi_1 z_1 - \psi_2 z_2 - \psi_3 z_3 - \psi_4 z_4 - \psi_5 z_5] \end{aligned} \quad \textbf{(3.36)}$$

The stability of the closed loop system using controller (4.36) can be checked by using the following Lyaponuv equation:

$$V_{a_4} = V_4 + \frac{1}{2}\xi_4^2 \quad \textbf{(3.37)}$$

The derivative of the lyapunov function (4.37) is as follows:

$$\dot{V}_{a_4} = \dot{V}_4 + \dot{\xi}_4 \xi_4 \leq 0 \quad \textbf{(3.38)}$$

3.3 Simulation results

The performance of the closed loop system is simulated using the MATLAB software and the results are plotted for the states with initial values =[0.2 0 0 0 0] and k=1. Figure 8 – Figure 12 show the plots of $\phi,\ p,\ \delta,\ \beta,\ \frac{\partial \beta}{\partial t}$ respectively.

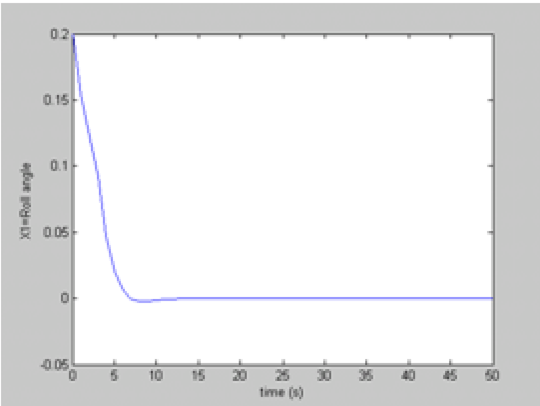


Figure8 Roll angle (rad.)

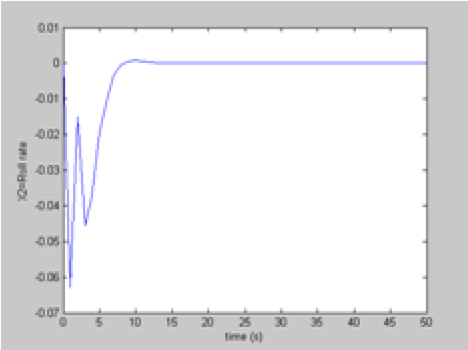


Figure 9 Roll-rate (rad/sec)

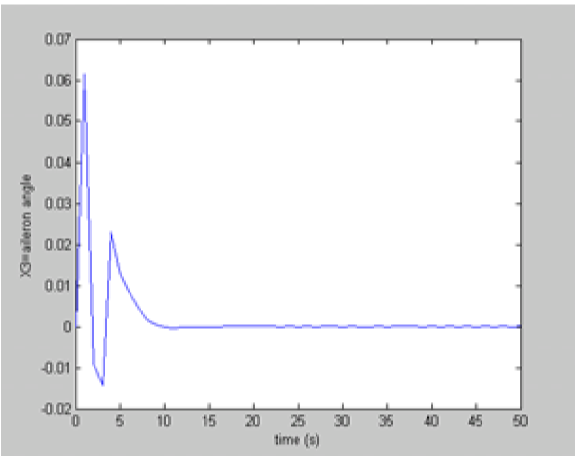


Figure 10 Aileron deflection angle (rad.)

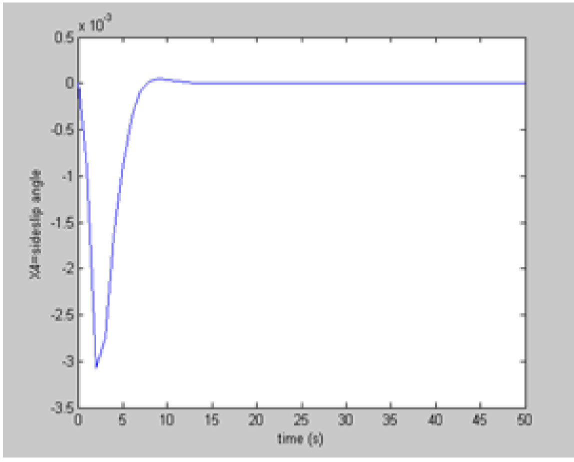


Figure 11 sideslip angle (rad.)

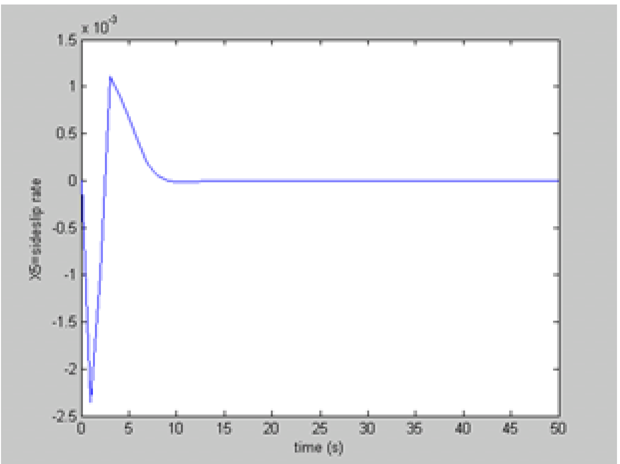


Figure 12 sideslip rate (rad./sec)

IV. Conclusion

The above figures show that all the states of the system converge to zero asymptotically. Hence, it can be concluded that the proposed backstepping controller works well for the stabilization of the wing rock phenomenon.

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